Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name : Engineering Mathematics-I

Subject Code : 4TE01EMT1 **Branch: B.Tech (All)**

Semester : 1 **Date : 28/11/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

1. Use of Programmable calculator & any other electronic instrument is prohibited.
2. Instructions written on main answer book are strictly to be obeyed.
3. Draw neat diagrams and figures (if necessary) at right places.
4. Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $z = 1+\sqrt{3}\,i$ then $\mathrm{Re}(z) =$ ______.

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $1-\sqrt{3}\,i$

b) The polar form of $z = 1+i$ is ______.

(a) $\sqrt{2}e^{\frac{3\pi}{4}i}$ (b) $\sqrt{2}$ (c) $\sqrt{2}e^{\frac{\pi}{4}i}$ (d) $\sqrt{2}e^{-\frac{\pi}{4}i}$

c) $i^4 =$ _______.

(a) 1 (b) -1 (c) i (d) $-i$

d) $\lim\limits_{x\to 0}\dfrac{\tan x}{3x} =$ _____.

(a) 3 (b) $\dfrac{1}{3}$ (c) 1 (d) 0

e) $\lim\limits_{h\to 0}\left(\dfrac{2^h-1}{h}\right) =$ ______.

(a) log2 (b) log$e$ (c) $\log 1$ (d) $\dfrac{1}{e}$

f) $\lim\limits_{h\to 0}\dfrac{\cos(x+h)-\cos x}{h} =$ _____.

(a) $\cos x$ (b) $\sin x$ (c) $-\cos x$ (d) $-\sin x$

g) The series $\sum \dfrac{1}{n^k}$ is convergent if

(a) $k=1$ (b) $k>1$ (c) $k<1$ (d) none of these

h) The series $\sum(-1)^n$ is

(a) convergent (b) divergent (c) oscillatory (d) none of these

i) If the curve $x^3+y^3=3axy$ is symmetrical about ______.

(a) X-axis (b) Y-axis (c) both X and Y axes (d) none of these

j) The series $x+\frac{x^3}{3!}+\frac{x^5}{5!}+....$ represent expansion of

(a) $\sin x$ (b) $\cos x$ (c) $\sinh x$ (d) $\cosh x$

k) If $u = ax^2 + 2hxy + by^2$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to

(a) $2u$ (b) u (c) 0 (d) none of these

l) What is the value of $\frac{\partial}{\partial x}\left(x^y\right)=$ ______.

(a) x^y (b) $x^y \log x$ (c) yx^{y-1} (d) none of these

m) $\lim_{(x,y)\to(1,-2)} \frac{3x-4y}{x-y}=$ ______.

(a) $\frac{3}{11}$ (b) -5 (c) 5 (d) none of these

n) If $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\partial x}{\partial r}$ is equal to

(a) $\sec\theta$ (b) $\operatorname{cosec}\theta$ (c) $\cos\theta$ (d) $\sin\theta$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find all n^{th} roots of unity. (05)

b) Using De Moivre's theorem prove that $\cos^4\theta=\frac{1}{8}\left(\cos 4\theta+4\cos 2\theta+3\right)$. (05)

c) Simplify: $\dfrac{\left(\cos 5\theta - i\sin 5\theta\right)^{\frac{2}{5}}\left(\cos\frac{2\theta}{5}+i\sin\frac{2\theta}{5}\right)^5}{\left(\cos 3\theta + i\sin 3\theta\right)^{\frac{1}{3}}\left(\cos\frac{2\theta}{3}-i\sin\frac{2\theta}{3}\right)^{18}}$ (04)

Q-3 **Attempt all questions** **(14)**

a) Trace the Cissoids $y^2\left(2a-x\right)=x^3$. (05)

b) Expand $f\left(x\right)=\sin x$ in powers of x up to x^5 by Maclaurin's series. (05)

c) Find the modulus, principal argument, complex conjugate and polar form of $z=-1+i$. (04)

Q-4 **Attempt all questions** **(14)**

a) Discuss whether the function $f\left(x\right)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 0 & x=0\end{cases}$ is continuous at $x=0$. (05)

b) Evaluate: $\lim_{x\to 0}\left(\frac{\tan x-\sin x}{x^3}\right)$ (05)

c) If $f\left(x\right)=\begin{cases}1+x, & if\ x\leq 2\\ 5-x, & if\ x>2\end{cases}$ then show that $f'\left(2\right)$ does not exist. (04)

Q-5		**Attempt all questions**	**(14)**
	a)	Test for convergence the series i) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ ii) $\sum_{n=1}^{\infty} \frac{1}{2^n}$	(05)
	b)	Find radius of convergence and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{2n-1}$.	(05)
	c)	Define: $\frac{\partial(u,v)}{\partial(x,y)}$. If $x = r\cos\theta$ and $y = r\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.	(04)
Q-6		**Attempt all questions**	**(14)**
	a)	If $z = yf(x^2 - y^2)$ then show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = \frac{xz}{y}$.	(05)
	b)	If $f(x,y) = \log(y\sin x + x\sin y)$ then find $\frac{\partial^2 f}{\partial x\partial y}, \frac{\partial^2 f}{\partial y\partial x}$.	(05)
	c)	Find the equation of tangent plane and normal line to the surface $x^2 + 2y^2 + 3z^2 = 12$ at the point $(1,2,-1)$.	(04)
Q-7		**Attempt all questions**	**(14)**
	a)	Find the maximum and minimum values of $f(x,y) = 2 + 2x + 2y - x^2 - y^2$.	(05)
	b)	Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at $(0,0)$.	(05)
	c)	Expand e^x in powers of $(x+3)$.	(04)
Q-8		**Attempt all questions**	**(14)**
	a)	Define Homogeneous function and State Euler's theorem and using it find i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ and ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ if $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$.	(07)
	b)	Trace the curve $r = a(1 - \cos\theta)$.	(07)